



# ASQ CRE Prep course

Lesson II. A. 4. d.

Poisson Process Models

Mann Reverse Arrangement Test

Looking for trends in time to failure data

# MANN REVERSE ARRANGEMENT TEST



# What is a Reversal?

- **A reversal is an interarrival time that is less than a time that occurs later.**
- **An example 50, 145, 232, 120, 150**  
**50 is less than all 4, thus is 4 reversals**  
**145 is less than 232 and 150, for 2**  
**232 is greater than remaining, thus 0**  
**120 is less than 150, for 1**  
**7 total reversals in this data set**

# Step 1: Setup Hypothesis

- **The null hypothesis:**  
**Times are independent and identically distributed**
- **The alternative hypothesis (pick one):**  
 **$H_{11}$  decreasing interarrival times**  
 **$H_{12}$  increasing interarrival times**  
 **$H_{13}$  non-constant interarrival times**

## **Step 2: Form sequence of inter-arrival times**

- **Let's use the example of:**

**36, 63, 86, 128, 165, 324**

**are days on which repairs occur  
repairs take about an hour, this is days  
from start of operation, not interarrivals**

# Step 3: Count Reversals

- **Given the data count reversals of interarrival time (days)  
Convert data to interarrivals**

**36, 27, 23, 42, 37, 159**

- **Counting reversals, I get**

$$\mathbf{R = 3 + 3 + 3 + 1 + 1 = 11}$$

# Step 4: Compute Mean and Std

- **Mean**

$$\text{Mean } \mu_R = \frac{n(n-1)}{4} = \frac{11(11-1)}{4} = 27.5$$

- **Standard Deviation**

$$\text{Stdev } \sigma_R = \frac{\sqrt{n(n-1)(n+2.5)}}{6} = \frac{\sqrt{11(11-1)(11+2.5)}}{6} = 6.42$$

# Step 5: Set critical value

- **For a 0.05 significance level the z-value is**

$$Z_{0.05} = 1.645, \text{ and } Z_{0.025} = 1.96$$

- **$H_{11}$  decreasing interarrival times**

$$\text{Conclude } H_{11} \text{ if } R \leq r_{1-\alpha} \approx \text{Ceiling} \left[ \mu_r - z_{\alpha} \sigma_R - c_{\alpha} \right] = \frac{n(n-1)}{2} - r_{\alpha}$$

- **$H_{12}$  increasing interarrival times**

$$\text{Conclude } H_{12} \text{ if } R \geq r_{\alpha} \approx \text{Floor} \left[ \mu_r + z_{\alpha} \sigma_R + c_{\alpha} \right] = \frac{n(n-1)}{2} - r_{\alpha-1}$$

- **$H_{13}$  non-constant interarrival times**

$$\text{Conclude } H_{13} \text{ if } R \leq r_{1-\alpha/2} \text{ or } R \geq r_{\alpha/2}$$

# Z and c values

$\alpha$	0.1	0.05	0.025	0.01
$z_{\alpha}$	1.28155	1.64485	1.95996	2.32635
$c_{\alpha}$	1.62	1.48	1.34	1.09

$z_{\alpha}$  is from the standard normal table

$c_{\alpha}$  is a correction to the calculation when n is less than 30

# Step 6: Compare Count to Critical

- Is the process getting more reliable?

Conclude  $H_{12}$  if  $R \geq r_\alpha$

$$r_\alpha \approx \text{Floor} [\mu_r + z_\alpha \sigma_R + c_\alpha]$$

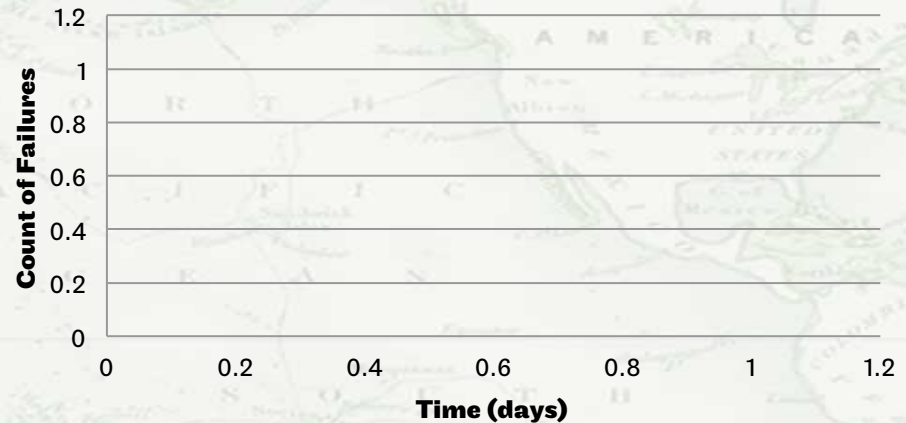
$$r_\alpha \approx \text{Floor} [27.5 + 1.645(6.42) + 1.48]$$

$$r_\alpha \approx 39$$

$$R = 11$$

$R \leq r_\alpha$ , therefore no evidence to say it's improving

**Mean Cumulative Plot**



Do you remember  
Hypothesis  
Testing?



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Poisson Process Models

Laplace's Trend Test