



ASQ CRE Prep course

Lesson II. A. 4. d.

Poisson Process Models

Mann Reverse Arrangement Test

A wide-angle photograph of a sunset over a body of water. The sky is filled with warm, orange and yellow hues near the horizon, transitioning to cooler blues and purples higher up. Silhouettes of hills or mountains are visible on the left, and the water in the foreground has small, glistening waves reflecting the light.

Looking for trends in time to failure data

MANN REVERSE ARRANGEMENT TEST

What is a Reversal?

- A reversal is an interarrival time that is less than a time that occurs later.
- An example 50, 145, 232, 120, 150
50 is less than all 4, thus is 4 reversals
145 is less than 232 and 150, for 2
232 is greater than remaining, thus 0
120 is less than 150, for 1
7 total reversals in this data set

Step 1: Setup Hypothesis

- **The null hypothesis:**
Times are independent and identically distributed
- **The alternative hypothesis (pick one):**
 H_{11} **decreasing interarrival times**
 H_{12} **increasing interarrival times**
 H_{13} **non-constant interarrival times**

Step 2: Form sequence of inter-arrival times

- Let's use the example of:

36, 63, 86, 128, 165, 324

**are days on which repairs occur
repairs take about an hour, this is days
from start of operation, not interarrivals**

Step 3: Count Reversals

- Given the data count reversals of interarrival time (days)
Convert data to interarrivals

36, 27, 23, 42, 37, 159

- Counting reversals, I get

$$R = 3 + 3 + 3 + 1 + 1 = 11$$

Step 4: Compute Mean and Std

- **Mean**

$$\text{Mean } \mu_R = \frac{n(n-1)}{4} = \frac{11(11-1)}{4} = 27.5$$

- **Standard Deviation**

$$\text{Stdev } \sigma_R = \frac{\sqrt{n(n-1)(n+2.5)}}{6} = \frac{\sqrt{11(11-1)(11+2.5)}}{6} = 6.42$$

Step 5: Set critical value

- **For a 0.05 significance level the z-value is**

$$Z_{0.05} = 1.645, \text{ and } Z_{0.025} = 1.96$$

- **H_{11} decreasing interarrival times**

$$\text{Conclude } H_{11} \text{ if } R \leq r_{1-\alpha} \approx \text{Ceiling} \left[\mu_r - z_\alpha \sigma_R - c_\alpha \right] = \frac{n(n-1)}{2} - r_\alpha$$

- **H_{12} increasing interarrival times**

$$\text{Conclude } H_{12} \text{ if } R \geq r_\alpha \approx \text{Floor} \left[\mu_r + z_\alpha \sigma_R + c_\alpha \right] = \frac{n(n-1)}{2} - r_{\alpha-1}$$

- **H_{13} non-constant interarrival times**

$$\text{Conclude } H_{13} \text{ if } R \leq r_{1-\alpha/2} \text{ or } R \geq r_{\alpha/2}$$

Z and c values

α	0.1	0.05	0.025	0.01
z_α	1.28155	1.64485	1.95996	2.32635
c_α	1.62	1.48	1.34	1.09

z_α is from the standard normal table

c_α is a correction to the calculation when n is less than 30

Step 6: Compare Count to Critical

- **Is the process getting more reliable?**

Conclude H_{12} if $R \geq r_\alpha$

$$r_\alpha \approx \text{Floor} [\mu_r + z_\alpha \sigma_R + c_\alpha]$$

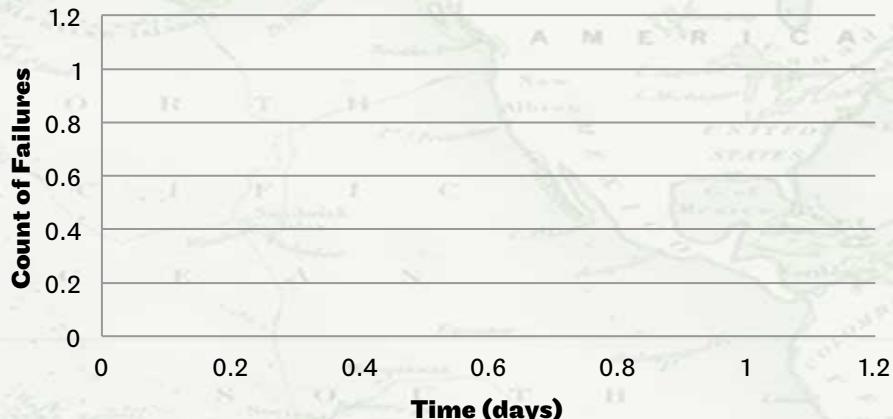
$$r_\alpha \approx \text{Floor} [27.5 + 1.645(6.42) + 1.48]$$

$$r_\alpha \approx 39$$

$$R = 11$$

$R \leq r_\alpha$, therefore no evidence to say it's improving

Mean Cumulative Plot



Do you remember
Hypothesis
Testing?



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Poisson Process Models

Laplace's Trend Test